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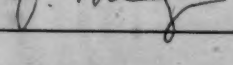

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THE UNIVERSITY OF ALBERTA

EXPECTATIONS, INDEXATION & MONETARY POLICY

BY

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

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DEPARTMENT .....ECONOMICS.....

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled .....

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submitted by ....CHEN.P..HUANG.....

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A B S T R A C T

An integrated model which combines the Forster-Gupta model, the Ferguson-Gupta model, and the Taylor model is presented to examine (1) the effect of money illusion on the stability of the model; and (2) the effect of a change in the rate of monetary growth on the economy.

Also, the effects on economic stability of indexing the tax system and government bond yields for inflation is re-examined by modifying the Scarth model to assume rational expectations.

The study shows (1) money illusion works as a stabilizing factor in the economy; (2) in the long run, an increase in the rate of monetary growth will produce an increase in the inflation rate and a decrease in the real rate of interest; and (3) indexing the tax system and government bond yields for inflation makes the model more unstable.

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I. INTRODUCTION

Inflation is an economic phenomenon characterized by a continuous rising of prices, or equivalently, by a continuous fall in the value of real money. Unemployment is also an economic phenomenon characterized by rising rates of structural and frictional unemployment. According to Phillips [1] there is a short-run tradeoff between inflation and the unemployment rate, that is, when the rate of inflation decreases (increases) the unemployment rate increases (decreases).

Monetary policy is an economic tool used by the authorities to achieve certain economic goals. Monetary policy is different from fiscal policy in the mechanism or channels through which the policy exercises its effects on the economy. The two most important channels are through its impact on credit availability or money supply and on the interest rate.

Friedman [2], in his famous article "The role of Monetary Policy", argued that there are three main things which monetary policy can do. First, monetary policy can prevent money itself from being a major source of economic disturbance. Secondly, monetary policy can provide a stable background for the economy. And finally, monetary policy can contribute to offsetting major disturbances in the economic system arising from other sources.

Money serves three functions in the economy; as means of exchange, as a store of value, and as a unit of account. When the

value of money falls due to inflation, the three functions of money will gradually be undermined. Hence, inflation is not desirable from an economic point of view.

To understand how monetary policy works as an instrument to cure inflation, we can examine the effect of changes in the rate of money supply on the inflation rate. Also, there is a short-sighted way to fight some of the effects of inflation: indexation. The reason that indexation is short-sighted is because it does nothing to the rate of inflation. It simply sets up a screen to insulate the economy from disturbances so that looking from outside of the screen the effect of inflation seems vanished.

In his study of hyperinflation Cagan [3] presented a model which introduced an adaptive inflationary expectations process and a demand for real money balances as a function of the expected rate of inflation. The stability of the equilibrium was found to depend on the structural parameters of the model.

Vanderkamp [4] dropped the adjustment equation for inflationary expectations and introduced an expectations-augmented Phillips curve, an adjustment equation for the unemployment rate, and a demand for real balances function dependent only on real income. The equilibrium of the model was implicitly assumed stable.

Taylor [5] developed a dynamic model which integrated the Keynesian IS-LM model with the Friedman-Phelps natural rate hypothesis and the Cagan adaptive expectations model to examine the effect of a change in the rate of monetary growth in the economy. He found that

in the short run, an increase in the rate of monetary expansion produces a decline in the interest rate and a higher level of real output. But, in the long run, there is no effect on real output, and both the rate of inflation and the nominal rate of interest increase by the amount of increase in the rate of monetary growth.

Forster and Gupta [6] presented a synthetic model which modified the Vanderkamp model [4] by employing a Phillips curve which allows for the existence of money illusion. By examining the dynamic properties of the synthetic model, they analyzed the effectiveness of Friedman-type monetary rule as against discretionary monetary policies in stabilizing the economy. They found that if Cagan's condition is satisfied, equilibrium of the model is stable. Hence, Friedman-type monetary rule will be effective. However, if the equilibrium is unstable then discretionary monetary policy can be used to obtain stability.

Ferguson and Gupta [7] further modified the Forster-Gupta model by introducing disequilibrium in the money market and by using a form of monetarist adjustment process to link money market disequilibrium to real income à la Black [8]. They found that Cagan's condition is sufficient to ensure the stability of the model. Also, the money illusion acts as a stabilizing factor. The greater the degree of money illusion, the less inflation will increase and the smaller the increase in the expected rate of inflation. As for policy implications, they reached the same conclusions as the earlier study by Forster-Gupta.

A drawback of both the Forster-Gupta and Ferguson-Gupta models, is that they do not include the IS-schedule in their models. On the other hand, Taylor's model does not consider the effect of money illusion. To make the model complete, we shall develop an integrated model which combines all the features in the above-mentioned models. In Section II we modify the Forster-Gupta model to include the IS-schedule and examine the stability problem underlying the model. In section III, we modify the Ferguson-Gupta model to include the IS-schedule and again, analyze the stability and dynamic properties in the system.

Although it has been suggested that widespread indexation of wages and prices would eliminate or moderate the short-run tradeoff between output and unanticipated inflation, indexation can not change the rate of inflation. Gray [9] examined wage indexation in a macroeconomic model. She found that while indexing insulates the real sector from the effects of monetary shocks, it may exacerbate the real effects of real shocks. Gray's model does not consider the expected rate of inflation. When we modify the demand for money function in her model to incorporate the expected inflation rate, we find that unless there is a positive relationship between the money supply and the expected rate of inflation, and a negative relationship between the aggregate output and the expected rate of inflation. Gray's conclusions may no longer hold (See details in Appendix 1).

Barro [10] analyzed the effect of indexing on price and quantity determination in a market-clearing framework that incorporates rational formation of expectations. He concluded that as indexing does nothing to speed up the flow of information (the underlying element in the Phillips curve in this type of model), it has no effect on (the entire probability distribution of) output. Thus, the familiar hypothesis that indexing would moderate the Phillips curve is not supported by Barro's model.

Fisher [11] suggested that an indexed economy is likely to be more unstable with respect to real changes than a non-indexed one, with the converse holding with respect to monetary changes (same as Gray).

Scarth [12] used a standard macro model to examine the effects on economic stability of indexing the tax system and government bond yields for inflation. He concluded that financing through money creation feeds the inflation, and financing through bond sales increases the deficit in future periods. Both financing options represent explosive processes. He further claimed that his model should be regarded as a formal extension of the system that Friedman has discussed to defend his policy advice that pegging the rate of interest makes the economy unstable [2].

To investigate the stability problems of indexation further, we shall examine Scarth's model under a different theory of expectations formation, namely, rational formation of expectations, in

Section IV of this thesis.

Finally, general conclusions are drawn in Section V.

II. A SIMPLE MODEL

As mentioned before, this model integrates the Forster-Gupta model [6] and the Taylor model [5] to include both the IS-schedule and the money illusion parameter. The main findings are: (1) Cagan's stability condition is sufficient but not necessary for the system to be stable; (2) in the long run, increased monetary growth will result in an increase in the inflation rate, a decrease in the real rate of interest and a net change in the nominal interest rate. The extent of these changes depends upon the structural parameters of the model such as the degree of money illusion and the slope of the IS curve. In other words, with a less than vertical Phillips curve, a higher inflation rate produces a higher steady state level of real income and simultaneously a lower real rate of interest.

A. The Model

1. $\ln Y = \alpha - \mu R, \quad \mu > 0$
2. $\ln (M/PY) = \phi - \lambda i, \lambda > 0$
3. $R = i - \pi^*$
4. $D\pi^* = \delta(\pi - \pi^*), \delta > 0$
5. $\pi = a_0 + a_1 U + g\pi^*, a_0 > 0, a_1 < 0$
6. $dU/dt = b(y - y^*), b < 0$

Equation (1) is the IS curve, (2) is the LM curve, (3) is the Fisher relationship between nominal and real interest rates, (4) is the adaptive expectations hypothesis, (5) is the

augmented Phillips curve, and (6) describes the dynamic behavior of the unemployment deviation. The symbols are defined as follows: M is money supply, $m \equiv (1/M)(dM/dt)$, R is real rate of interest, i is nominal interest rate, P is the price level, $\pi \equiv (1/P)(dP/dt)$, π^* is the expected rate of inflation, U is the unemployment rate, Y is real income, y is the rate of growth of real income, y^* is the trend rate of growth of real income, D is the derivative with respect to time, and g is the money illusion parameter.

From (1), $D\ln Y = -\mu DR$

From (2), $\pi - \lambda Di - \mu DR = m$ (2')

Also, from (1) $\ln V = \alpha - \mu\rho$ where V is the long run level of real income and ρ is the long run level of real rate of interest.

From (5) and (6), note that $y^* = (\frac{1}{V})(\frac{dV}{dt})$

$b(y-y^*) = b(D\ln Y - D\ln V) = \frac{1}{a_1} D(\pi - g\pi^*)$

$D\ln\frac{Y}{V} = \frac{1}{a_1 b} D(\pi - g\pi^*)$

$\ln\frac{Y}{V} = \frac{1}{a_1 b}(\pi - g\pi^*)$

Set $\frac{1}{a_1 b} = \beta$, the above equation becomes

$$\ln Y = \ln V + \beta(\pi - g\pi^*)$$

make substitutions,

$$\alpha - \mu R = \alpha - \mu\rho + \beta\pi - \beta g(i - R)$$

Rearrange terms, we finally have

$$\beta\pi - \beta g i + (\beta g + \mu) R = \mu\rho \qquad (6')$$

From (4), $-\delta\pi + (\delta+D)\, i - (\delta+D)R = 0 \quad (4')$

Put (2'), (6') and (4') into matrix form:

$$\begin{pmatrix} 1 & -\lambda D & -\mu D \\ \beta & -\beta g & \beta g+\mu \\ -\delta & \delta+D & -(\delta+D) \end{pmatrix} \begin{pmatrix} \pi \\ i \\ R \end{pmatrix} = \begin{pmatrix} m \\ \mu\rho \\ o \end{pmatrix}$$

The determinant of the above coefficient matrix is

$\Delta = -\mu\delta - (\mu\beta\delta - \delta\beta g\lambda + \beta\delta\lambda - \delta\mu\lambda + \mu - \delta\beta g\mu)D - \beta(\mu+\lambda)D^2$

We consider two extreme cases: no money illusion, that is, g=1, and complete money illusion, g=0. First, g=1. For the system to be stable, the roots of the characteristic equation $\Delta=0$ must be negative which means $-\delta\mu\lambda + \mu > o$ or $\delta\lambda < 1$. This is exactly Cagan's stability condition.

Next, let's consider g=0. Stability condition requires that

$\mu\beta\delta + \lambda\beta\delta - \mu\lambda\delta + \mu > o$

or $\lambda\delta < 1 + \beta\delta + \frac{\lambda}{\mu}\,\beta\delta$

Since $\beta, \delta, \lambda, \mu > o$, it is easy to see that if Cagan's condition $\delta\lambda<1$ is satisfied, stability is assured. However, the reversed statement is not true. Thus, Cagan's stability condition is sufficient in this case. In the case that $o<g<1$, we can easily see that Cagan's condition is sufficient and not necessary.

B. The Dynamic Nature of the Model

For a step change in the rate of monetary growth at time t=o, say $\bar{m}$, the response of the inflation rate, the nominal interest rate, and the real interest rate for time $t > o$ can be solved using Laplace transforms [13]:

$$\pi = \pi_o + \left(1 + \frac{\theta_2 - \theta_3}{\theta_1 - \theta_2} e^{-\theta_1 t} + \frac{\theta_3 - \theta_1}{\theta_1 - \theta_2} e^{-\theta_2 t}\right) \bar{m}$$

$$i = i_o + \left(1 - \frac{(1-g)\beta}{\mu} + \frac{\theta_2\left(1 - \frac{(1-g)\beta}{\mu}\right) - \theta_4}{\theta_1 - \theta_2} e^{-\theta_1 t} + \right.$$

$$\left. \frac{\theta_4 - \left(1 - \frac{(1-g)\beta}{\mu}\right)\theta_1}{\theta_1 - \theta_2} e^{-\theta_2 t}\right) \bar{m}$$

$$R = R_o + \left(-\frac{(1-g)\beta}{\mu} + \frac{-\frac{(1-g)\beta}{\mu}\theta_2 - \theta_4}{\theta_1 - \theta_2} e^{-\theta_1 t} + \right.$$

$$\left. \frac{\frac{(1-g)\beta}{\mu}\theta_1 + \theta_4}{\theta_1 - \theta_2} e^{-\theta_2 t}\right) \bar{m}$$

where π_o, i_o, R_o are the initial values at $t = o$, θ_1 and θ_2 are the roots of the characteristic equation mentioned in sub-section (A), $\theta_3 = \frac{\mu}{\beta(\mu+\lambda)}$ and $\theta_4 = -\frac{1}{\mu+\lambda}$.

The detailed derivations of the above three equations can be found in Appendix 2.

In the long run, when $t \to \infty$, we have

$$\pi = \pi_o + \bar{m}$$

$$i = i_o + [1 - (1-g)\beta/\mu]\,\bar{m}$$

$$R = R_o + [-(1-g)\beta/\mu]\,\bar{m}$$

Thus, in the case of an increase of monetary growth $\bar{m}$ the inflation rate will increase by $\bar{m}$ and the real interest rate will decrease by $[(1-g)\beta/\mu]\,\bar{m}$.

C. A Special Case

An alternative assumption to adaptive expectations is rational expectations. In a deterministic model, rational expectations are equivalent to perfect foresight, that is, $\pi^* = \pi$.

Substitute into (3), $R = i - \pi$

From (2') $\pi - \lambda D(R+\pi) - \mu DR = m$

or $(1-\lambda D)\pi - (\lambda D+\mu D)R = m$ (2")

From (6') $\beta\pi - \beta g\pi + \mu R = \mu\rho$

or $\beta(1-g)\pi + \mu R = \mu\rho$ (6")

Put (2"), (6") into matrix form

$$\begin{pmatrix} 1-\lambda D & -(\lambda D+\mu D) \\ \beta(1-g) & \mu \end{pmatrix} \begin{pmatrix} \pi \\ R \end{pmatrix} = \begin{pmatrix} m \\ \mu\rho \end{pmatrix}$$

The determinant of the above coefficient matrix is

$$\Delta = \mu(1-\lambda D) + \beta(1-g)\ (\lambda D + \mu D)$$

Stability condition requires that the root of the characteristic equation be negative, that is:

$$\frac{\mu}{\mu\lambda - \beta(1-g)\ (\lambda+\mu)} < 0$$

or $$\mu\lambda - \beta(1-g)\ (\lambda+\mu) < 0$$

If $g=1$, that is, there is no money illusion, the above inequality cannot be true. Thus, the model is unstable. However, if $g<1$, the model can be stable if $\mu\lambda<\beta(1-g)\ (\lambda+\mu)$. We conclude that the money illusion parameter acts as a stabilizing factor in the model.

III. A DYNAMIC MODEL

In this section, the simple model used in Section II is modified to introduce disequilibrium in the money market à la Black [8] and Ferguson-Gupta [7] by using a form of monetarist adjustment process to link money market disequilibrium to real income again in the spirit of Black and Ferguson-Gupta.

The Model

1. $\ln Y = \alpha - \mu R, \quad \mu > 0$
2. $\ln(M/PY) = \phi - \lambda i, \quad \lambda > 0$
3. $R = i - \pi^*$
4. $D\dot{\pi}^* = \delta(\pi - \pi^*), \quad \delta > 0$
5. $\pi = a_0 + a_1 U + g\pi^*, \quad a_0 > 0, \quad a_1 < 0$
6. $\dfrac{dU}{dt} = b\,(y - y^*), \quad b < 0$
7. $y - y^* = \theta(n - m), \quad \theta > 0$

Symbols are defined as in Section II. The model is the same as the simple model except here a new equation (7) being introduced where y is the rate of growth of real income, $y \equiv (\frac{1}{Y})(\frac{dY}{dt})$, y^* is the trend rate of growth of real income, $m \equiv (\frac{1}{M})(dM/dt)$ is the growth rate of the demand for money, and $n \equiv (\frac{1}{N})(\frac{dN}{dt})$ is the growth rate of the supply of money (N, the money supply is assumed to be exogenous).

From (1), $D\ln Y = -\mu DR$

From (2), $m = \pi - \lambda Di - \mu DR$ (2')

From (4), $-\delta\pi + (\delta + D)i - (\delta+D)R = o$ (4')

From (5), (6) and (7),

$$\frac{dU}{dt} = b\theta(n-m) = \frac{1}{a_1} D(\pi - g\pi^*)$$

$$n-m = \frac{1}{a_1 b\theta} D(\pi - g\pi^*)$$

Set $\frac{1}{a_1 b\theta} = \beta$

$$n-m = \beta D(\pi - gi + gR) \quad \text{(From (3), } \pi^* = i-R\text{)}$$

$$= \beta D\pi - \beta gDi + \beta gDR \quad (6')$$

Put (2'), (4') and (6') into matrix notation:

$$\begin{pmatrix} 1 & -\lambda D & -\mu D \\ \beta D & -\beta gD & \beta gD \\ -\delta & \delta+D & -(\delta+D) \end{pmatrix} \begin{pmatrix} \pi \\ i \\ R \end{pmatrix} = \begin{pmatrix} m \\ n-m \\ o \end{pmatrix}$$

The determinant of the above matrix is

$$\Delta = \beta gD(\delta+D) + \lambda\delta\beta gD^2 - \mu\beta(\delta+D)D^2 + \mu\delta\beta gD^2 - \lambda\beta(\delta+D)D^2 - \beta gD(\delta+D)$$

$$= D^2 [\lambda\delta\beta g - \mu\beta(\delta+D) + \mu\delta\beta g - \lambda\beta(\delta+D)]$$

The only non-zero root of the characteristic equation $\Delta = o$ is

$$\theta_1 = \frac{\lambda\delta\beta g - \mu\beta\delta + \mu\delta\beta g - \lambda\beta\delta}{\beta(\mu+\lambda)}$$

Stable condition requires $\lambda\delta\beta g - \mu\beta\delta + \mu\delta\beta g - \lambda\beta\delta < o$ or

$$\delta\beta(\mu+\lambda)(g-1) < o$$

If there is no money illusion so that $g = 1$, the model is not stable, however, if money illusion exists so that $g < 1$, the model is always stable. Thus, we can see that money illusion factor acts as a stabilizing factor in the model.

IV. AN INDEXING MODEL

Scarth [12] discussed the effects on economic stability of indexing the tax system and government bond yields for inflation using a model which assumes people adjust their expectations adaptively. In this section, we develop a model which assumes rational expectations. That is, the expected rate of price inflation is assumed to be equal to the rate of price inflation forecasted by the model.

A. The Model

$$Y = C(Y^d, r-\pi, W) + G \tag{1}$$

$$Y^d = (1-X)\,[Y + {}^{B}/_{P}] + {}^{XE}/_{P} \tag{2}$$

$${}^{M}/_{P} = L\,(Y, r, W) \tag{3}$$

$$W = {}^{M}/_{P} + {}^{B}/_{rP} \tag{4}$$

$$({}^{1}/_{P})({}^{dP}/_{dt}) = a\,[Y - Y^f]/Y^f \tag{5}$$

$$\pi = ({}^{1}/_{P})({}^{dP}/_{dt}) \tag{6}$$

$$PG + B = X[PY + B - E] + ({}^{dM}/_{dt}) + (\tfrac{1}{r})({}^{dB}/_{dt}) \tag{7}$$

The following notation is used:

- Y total factor income in real terms
- Y^d disposable income in real terms
- C total private spending (consumption and investment) in real terms
- G government spending in real terms
- Y^f full-employment or capacity output (exogenously determined)

B government bonds outstanding

M nominal money stock

P price level ($^{dP}/dt$ is the rate of change in price)

W real value of financial wealth

L demand of money

r nominal interest rate

π expected rate of inflation

X marginal tax rate

E nominal exemption level, defining income which is free from tax.

The assumptions about the partial derivatives (indicated by subscripts) and the parameters are as follows :

$C_3, L_1, a>0; C_2, L_2 < 0; 0<C_1, L_3, X<1$

B. Stability analysis of the model without indexing

Taking linear approximation to the model about full equilibrium we have:

$$d_1'(Y-Y^*) = d_2(r-r^*) + d_4(M-M^*) + d_5(B-B^*) + d_6(P-P^*) \qquad (L1)$$

$$d_7(M-M^*) = d_8(Y-Y^*) + d_9(r-r^*) + d_{10}(B-B^*) + d_{11}(P-P^*) \qquad (L2)$$

$$(^{dP}/dt) = (^{aP}/Y^f)\ (Y-Y^*) \qquad (L3)$$

$$(^{dM}/dt) + (^{1}/r)(^{dB}/dt) = -XP(Y-Y^*) + (1-X)(B-BB^*) + (G-XY)(P-P^*) \qquad (L4)$$

where

$d_1' = 1-C_1(1-X) + C_2(^{a}/r^f) \gtrless 0$

$d_2 = C_2 - C_3B/(Pr^2) < 0$

$d_4 = C_3/P > o$

$d_5 = C_1(1-X)/P + C_3/rP > o$

$d_6 = (^{-1}/P^2)\{C_1[B(1-X)+EX] + C_3[M+(^B/r)]\} < o$

$d_7 = (1 - L_3)/P > o$

$d_8 = L_1 > o$

$d_9 = L_2 - L_3B/(Pr^2) < o$

$d_{10} = L_3/rP > o$

$d_{11} = (^1/P^2)\ [M(1-L_3) - L_3{}^B/r] \gtrless o$

(L1) X d_9 - (L2) x d_2

$(d_1'\ d_9+d_2d_8)(Y-Y^*) = (d_2d_7+d_4d_9)(M-M^*) + (d_5d_9-d_2d_{10})(B-B^*)$

$+ (D_6d_9-d_2d_{11})(P-P^*)$

Substitute into (L3) and (L4), we have two cases :

(i) Money financed

$$dP/dt = (^{aP}/Y^f)\ [e_1'(M-M^*) + e_2'\ (P-P^*)]$$

$$dM/dt = -XPe_1'(M-M^*) + (G-XY-XPe_2')(P-P^*)$$

where $e_1' = \dfrac{d_2d_7 + d_4d_9}{d_1'd9 + d_2d_8}$, $e_2' = \dfrac{d_6d_9 - d_2d_{11}}{d_1'd_9+ d_2d_8}$

In matrix form

$$\begin{pmatrix} dP/dt \\ dM/dt \end{pmatrix} = \begin{pmatrix} aPe_1'/r^f & aPe_2'/Y^f \\ -XPe_1' & G-XY-XPe_2' \end{pmatrix} \begin{pmatrix} M-M^* \\ P-P^* \end{pmatrix}$$

$$\Delta = (aPe_1'/Y^f)(G-XY-XPe_2') - (XPe_1')(aPe_2'/r^f) = \frac{aPe_1'}{Y^f}(G-XY) \quad (A1)$$

$$\text{trace} = aPe_1'/Y^f + G - XY - XPe_2' \quad (A2)$$

Stable conditions require $\Delta > o$ and trace $< o$.

(ii) Bond financed

$$dP/dt = (aP/Yf)\,[e_3'(B-B^*) + e_2'(P-P^*)]$$

$$dB/dt = r\,[-xP(e_3'(B-B^*) + e_2'(P-P^*)) + (1-X)(B-B^*) + (G-XY)(P-P^*)]$$

$$= r\,(1-X-XPe_3')(B-B^*) + r\,(G-XY-XPe_2')(P-P^*)$$

where $e_3' = \dfrac{d_5d_9 - d_2d_{10}}{d_1'd_9 + d_2d_8}$ $\qquad$ $e_2' = \dfrac{d_6d_9 - d_2d_{11}}{d_1'd_9 + d_2d_8}$

In matrix form

$$\begin{pmatrix} dP/dt \\ dB/dt \end{pmatrix} = \begin{pmatrix} aPe_3'/_rf & aPe_2'/_Yf \\ r(1-X-XPe_3') & r(G-XY-XPe_2') \end{pmatrix} \begin{pmatrix} B-B^* \\ P-P^* \end{pmatrix}$$

$$\Delta = \frac{aPe_3'r}{Y^f}(G-XY-XPe_2') - \frac{aPe_2'r}{Y^f}(1-X-XPe_3')$$

$$= \frac{aPe_3'r}{Y^f}(G-XY) - \frac{aPe_2'r}{Y^f}(1-X) \quad (B1)$$

$$tr = \frac{aPe_3'}{Y^f} + r\,(G-XY-Xpe_2') \quad (B2)$$

Stable conditions require that $\Delta>0$ and $tr<0$.

C. Stability Analysis of the model with indexing

(i) Tax Exemption only

(a) money financed case

Equations (2) and (7) become (2a) and (7a) when indexing on tax exemption:

$$Y^d = (1-X)[Y+B/P] + XE \qquad (2a)$$

$$PG + B = X[PY+B-PE] + (dM/dt) + (1/r)(dB/dt) \qquad (7a)$$

Taking linear approximation about full equilibrium, the equations can be reduced to

$$\begin{pmatrix} dP/dt \\ dM/dt \end{pmatrix} = \begin{pmatrix} aPe_1'/Y^f & aPe_2''/Y^f \\ -XPe_1' & G-XY+XE-XPe_2'' \end{pmatrix} \begin{pmatrix} M-M^* \\ P-P^* \end{pmatrix}$$

where $e_1' = \dfrac{d_2d_7 + d_4d_9}{d_1'd_9 + d_2d_8}$ $\qquad$ $e_2'' = \dfrac{d_6'd_9 - d_2d_{11}}{d_1'd_9 + d_2d_8}$

and $d_6' = (-1/p2)[C_1B(1-X)+C_3(M+B/r)] < 0 \quad |d_6'| < |d_6|$

Stability conditions require that $\Delta>0$ and $tr<0$, that is,

$$\Delta = ({}^{aPe_1'}/_{\gamma f})(G-XY+XE-XPe_2^{*}) - (-XPe_1')({}^{aPe_2''}/_{\gamma f})$$

$$= \frac{aPe_1'}{\gamma^f}(G-XY+XE) > o$$

$$tr = {}^{aPe_1'}/_{\gamma f} + G - XY + XE - XPe_2'' < o$$

Comparing with (A1) and (A2), since $G-XY < o$, we can see that the addition of the term XE makes the system more unstable .

(b) Bond financed case

Putting the reduced equations into matrix form:

$$\begin{pmatrix} {}^{dP}/_{dt} \\ {}^{dB}/_{dt} \end{pmatrix} = \begin{pmatrix} {}^{aPe_3'}/_{\gamma f} & {}^{aPe_2''}/_{\gamma f} \\ r(1-X-XPe_3') & r(G-XY+XE-XPe_2'') \end{pmatrix} \begin{pmatrix} B-B^{*} \\ P-P^{*} \end{pmatrix}$$

where $e_2'' = \dfrac{d_6' d_9 - d_2 d_{11}}{d_1' d_9 + d_2 d_8}$, $e_3' = \dfrac{d_5 d_9 - d_2 d_{10}}{d_1' d_9 + d_2 d_8}$

$$\Delta = ({}^{aPe_3' r}/_{\gamma f})(G-XY+XE-XPe_2'') - (\frac{aPe_2''r}{\gamma^f})(1-X-XPe_3')$$

$$= (\frac{aPe_3' r}{\gamma^f})(G-XY+XE) - (\frac{aPe_2''r}{\gamma^f})(1-X)$$

$$tr = \frac{aPe_3'}{Y^f} + r(G-XY+XE-XPe_2'')$$

Stability conditions require $\Delta>0$ and $tr<0$.

Again, comparing with (B1) and (B2), the addition of the term XE makes the system more unstable.

(ii) Both tax exemption and bond yields are indexed

Equations (2), (4) and (7) now become

$$Y^d = (1-X)(Y+B)+XE \qquad (2b)$$

$$W = M/P + B(r-\pi) \qquad (4b)$$

$$PG+PB = X[PY+PB-PE] + (dM/dt) + [1/(r-\pi)](dB/dt) \qquad (7b)$$

Taking linear approximation about full equilibrium, we have

$$d_1''(Y-Y^*) = d_2'(r-r^*) + d_4(M-M^*) + d_5'(B-B^*) + d_6'(P-P^*)$$

$$d_7(M-M^*) = d_8'(Y-Y^*) + d_9'(r-r^*) + d_{10}'(B-B^*) + d_{11}'(P-P^*)$$

$$(dP/dt) = (aP/Y^f)\,(Y-Y^*)$$

$$(dM/dt) + (1/(r-\pi))(dB/dt) = -XP(Y-Y^*) + P(1-X)(B-B^*)$$

where

$$d_1'' = 1-C_1(1-X) + C_2\frac{a}{Y^f} - C_3\, aB/[Y^f(r-\frac{a(Y-Y^f)}{Y^f})^2]$$

$$d_2' = C_2 - C_3B/(r-\frac{a(Y-Y^f)}{Y^f})2$$

$$d_4 = C_3/P$$

$$d_5' = C_1(1-X) + C_3/(r - \frac{a(Y-Y^f)}{Y^f})$$

$$d_6' = -C_3M/P^2$$

$$d_7 = (1-L_3)/P$$

$$d_8' = L_1 + \frac{L_3a}{Y^f}$$

$$d_9' = L_2 - L_3B/(r - \frac{a(Y-Y^f)}{Y^f})^2$$

$$d_{10}' = L_3/(r - \frac{a(Y-Y^f)}{Y^f})$$

$$d_{11}' = (1-L_3)M/P^2$$

(a) Money financed

$$\begin{pmatrix} dP/dt \\ dM/dt \end{pmatrix} = \begin{pmatrix} aPe_1''/Y^f & aPe_2'''/Y^f \\ -XPe_1'' & -XPe_2''' \end{pmatrix} \begin{pmatrix} M-M^* \\ P-P^* \end{pmatrix}$$

$$\text{where } e_1'' = \frac{d_2' d_7 + d_4 d_9'}{d_1'' d_9 + d_2' d_8'} \quad , \quad e_2''' = \frac{d_6' d_9' - d_2' d_{11}'}{d_1'' d_9 + d_2' d_8'}$$

also, $|d_2'| > |d_2|$, $|d_9'| > |d_9|$, $|d_6'| < |d_6|$, $|d_{11}'| > |d_{11}|$ and

$$|d_8'| > |d_8|$$

$$\Delta = \left(\frac{aPe_1''}{Y^f}\right)(-XPe_2''') - \left(\frac{aPe_2'''}{Y^f}\right)(-XPe_1'') = o$$

$$tr = \frac{aPe_1''}{Y^f} - XPe_2'''$$

Stability conditions require that $\Delta > o$ and $tr < o$. Compare with (A1), (A2). Since $G-XY < o$, we can see that the system becomes more unstable under indexation of both tax exemption and bond yields.

(b) Bond financed

$$\begin{pmatrix} dP/dt \\ dB/dt \end{pmatrix} = \begin{pmatrix} aPe_3''/Y^f & aPe_2'''/Y^f \\ -XP(r-\pi)e_3'' + (r-\pi)P(1-X) & -XP(r-\pi)e_2''' \end{pmatrix} \begin{pmatrix} B-B^* \\ P-P^* \end{pmatrix}$$

$$\text{where } e_2''' = \frac{d_6' d_9' - d_2' d_{11}'}{d_1'' d_9' + d_2' d_8'} \quad , \quad e_3'' = \frac{d_5' d_9' - d_2' d_{10}'}{d_1'' d_9' + d_2' d_8'}$$

$$\Delta = \left(\frac{aPe_3'''}{Y^f}\right)(-XP(r-\pi)e_2''') - \left(\frac{aPe_2'''}{Y^f}\right)(-XP(r-\pi)e_3'''+(r-\pi)P(1-X))$$

$$= -(r-\pi)\left(\frac{aPe_2'''}{Y^f}\right) P(1-X)$$

$$tr = \frac{aPe_3''}{Y^f} - XP(r-\pi)\, e_2'''$$

Comparing with (B1) and (B2) and noting that stability conditions require $\Delta > o$ and $tr < o$, which implies that $e_2''' < o$ and $e_3'' < o$. Now since $\frac{aPe_3''r}{Y^f}$ $(G-XY) > o$ and $G-XY < o$, we can see that the system becomes more unstable because the trace is now less negative and the determinant is less positive.

(D) Conclusion

As mentioned earlier in the Introduction, under the assumption of adaptive expectations, Scarth finds that within his macro-economic model indexation of tax exemption and bond yield makes the model more unstable. From the detailed discussion above, we arrived at the same conclusion under the assumption of rational expectations.

V. GENERAL CONCLUSIONS

We have examined the effect of changes in the rate of money supply on the inflation rate. We find that, in the long run, an increase in the rate of monetary growth will produce an increase in the inflation rate. Thus, if the authority tries to achieve the goal of lower inflation rate, any policy which induces an increase in the rate of money supply will eventually prove to be ineffective. However, in the case where there is money illusion, people will be better off because the level of real income will be higher due to the decrease in the real interest rate. On the other hand, if there is no money illusion at all, then the real income will stay the same. Realistically, people always have certain degree of money illusion.

We have also demonstrated that the money illusion parameter acts as a stabilizing factor in the economy. Fisher's [14] argument that perfect foresight in monetary models produces instability is justified in our model in the case of no money illusion. However, if money illusion exists, the model can be stable.

The issue of indexing the economy is also examined. We agree that indexation has no effect on output. However, indexing the wages can insulate the economy from monetary disturbance. As for real shocks, we suspect the validity of the argument that

indexation will exacerbate their effects. If the expected rate of inflation actually affects the demand for money then indexing the wages may also insulate the economy from real shocks. Stability under indexation is explored finally. Our model strengthens Scarth's conclusion that indexing the tax system and government bond yields for inflation makes the model more unstable.

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Appendix 1 - GRAY'S MODEL REVISED

The Model

1. $Y = \alpha G(L)$, $\alpha = 1 + \mu$

2. $M^S = \beta \bar{m}$, $\beta = 1 + \xi$

3. $M^D = kPY e^{-\lambda \pi^*}$, π^*: expected rate of inflation

4. $M^S = M^D$

5. $L^D = f(\frac{w}{\alpha})$, $w = W/P$, $f_{w/\alpha < o}$

6. $L^S = g(w)$, $g_{w>o}$

The notations are the same as in Gray [9]. The model is also the same except equation (3), where π^* has been included.

Response of the System to Shocks

I. Monetary shocks

Assume $\mu = o$. (1) and (5) become

$Y = G(L)$ (1')

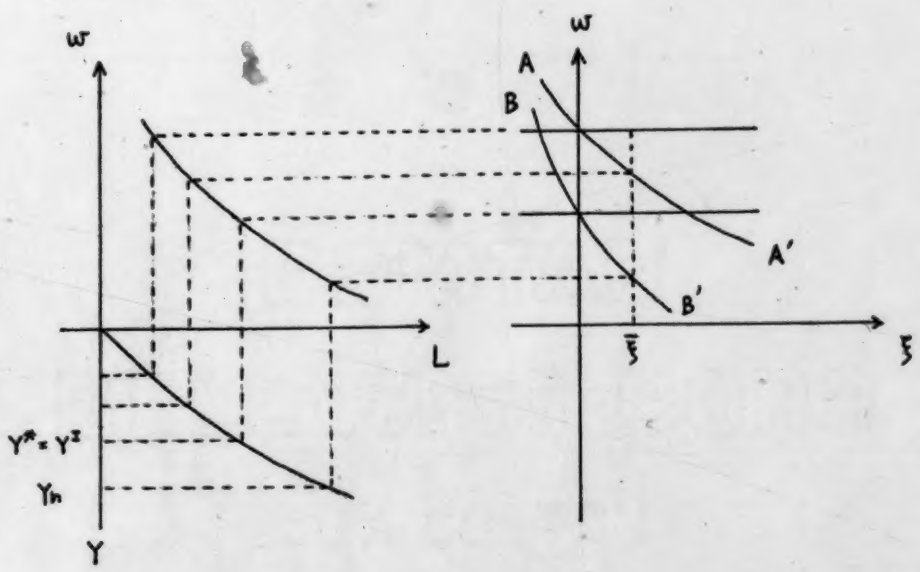
$L^D = f(w)$ (5')

From (2), (3) and (4)

$\beta \bar{M} = kPYe^{-\lambda \pi^*}$

or $(1+\xi)\ \bar{M} = k\ (W/w)\ G(f(w))e^{-\lambda\pi^*}$

In the non-indexing case, W is constant. If ξ increases, either w has to decrease or π^* has to decrease or both. But it is quite likely that π^* increases when ξ increases. Hence, w has to decrease drastically to compensate for the increase in π^*. Graphically, we have



AA': Gray's case

BB': Our case

BB' is steeper than AA'.

(II) Real shocks

Assume $\xi = o$, (2) becomes

$$M^S = \bar{M} \qquad (2)$$

From (2'), (3), and (4)

$$\bar{M} = kPYe^{-\lambda\pi^*}$$

or $$\bar{M} = k\left(W/w\right)(1+\mu)\, G(f(\frac{W}{1+\mu}))\, e^{-\lambda\pi^*}$$

In the non-indexing case W is constant. When μ increases either w or π^* has to increase or both have to increase.
Thus, the positive relationship between μ and w obtained by Gray may not hold because π^* can increase with w decreases when μ increases.

Appendix 2 - DERIVATIONS OF EQUATIONS

The matrix equation

$$\begin{pmatrix} 1 & -\lambda D & -\mu D \\ \beta & -\beta g & \beta g+\mu \\ -\delta & \delta+D & -(\delta+D) \end{pmatrix} \begin{pmatrix} \pi \\ i \\ R \end{pmatrix} = \begin{pmatrix} m \\ \mu\rho \\ o \end{pmatrix}$$

Can be solved using Cramer's rule:

$$\Delta'\pi = \mu(\delta+D)\, m \qquad (2a)$$

$$\Delta' i = \mu\delta\rho + (g\beta\delta-\beta\delta+\mu\delta-\beta D)\, m \qquad (2b)$$

$$\Delta' R = \mu\delta\rho - (\beta\delta-g\beta\delta+\beta D)\, m \qquad (2c)$$

where $\Delta' = \beta(\mu+\lambda)D^2+(\mu\beta\delta-\lambda g\beta\delta+\lambda\beta\delta-\lambda\mu\delta+\mu-\mu g\beta\delta)D+\mu\delta$

The right-hand side of the subsidiary equation eq. (2a) is

$$\frac{\mu(\delta+z)\, m}{z[\beta(\mu+\lambda)z^2+(\mu\beta\delta-\lambda g\beta\delta+\lambda\beta\delta-\lambda\mu\delta+\mu-\mu g\beta\delta)z+\mu\delta]}$$

Using partial fractions technique, we can put the above into

$$\frac{A(z+\theta_1)(z+\theta_2) + B(z+\theta_2)z + C(z+\theta_1)z}{z(z+\theta 1)(z+\theta_2)} \cdot m$$

where θ_1, θ_2 are the zeroes of the polynomial

$$z^2 + \frac{T}{\beta(\mu+\lambda)} z + \frac{\mu\delta}{\beta(\mu+\lambda)}$$

where $T = \mu\beta\delta - \lambda g\beta\delta + \lambda\beta\delta - \lambda\mu\delta + \mu - \mu g\beta\delta$

Compare the coefficients

$$A + B + C = o$$

$$A\theta_1 + A\theta_2 + B\theta_2 + C\theta_1 = \frac{\mu}{\beta(\mu+\lambda)}$$

$$A\theta_1 \quad \theta_2 = \frac{\mu\delta}{\beta(\mu+\lambda)}$$

Now, since $\theta_1\theta_2 = \frac{\mu\delta}{\beta(\mu+\lambda)}$ thus $A = 1$

Substitute and solve for B and C, we have

$$B = \frac{\theta_2 - \theta_3}{\theta_1 - \theta_2}$$

$$C = \frac{\theta_3 - \theta_1}{\theta_1 - \theta_2}$$

where $\theta_3 = \frac{\mu}{\beta(\mu+\lambda)}$

By standard forms of Laplace Transforms, we have

$$\pi = \pi_o + (1+Be^{-\theta_1 t}+Ce^{-\theta_2 t})\,\bar{m}$$

From the subsidiary equation of (2b) we get

$$\frac{(g\beta\delta-\beta\delta+\mu\delta-\beta z)\,m}{z[\beta(\mu+\lambda)z^2+Tz+\mu\delta]} = \frac{A(z+\theta_1)(z+\theta_2)+B(z+\theta_2)z+C(z+\theta_1)z}{z(z+\theta_1)(z+\theta_2)} \cdot m$$

by partial fractions.

Compare coefficients:

$$A + B + C = 0$$

$$A\theta_2+A\theta_1+B\theta_2+C\theta_1 = \frac{-1}{\mu+\lambda}$$

$$A\theta_1\theta_2 = \frac{g\beta\delta-\beta\delta+\mu\delta}{\beta(\mu+\lambda)}$$

Since $\theta_1\theta_2 = \frac{\mu\delta}{\beta(\mu+\lambda)}$ Thus $A = 1 - \frac{(1-g)\beta}{\mu}$

Substitute and solve for B and C:

$$B = \frac{\theta_2(1-\frac{(1-g)\beta}{\mu})\,\theta_4}{\theta_1 - \theta_2}$$

$$C = \frac{\theta_4-(1-\frac{(1-g)\beta}{\mu})\theta_1}{\theta_1 - \theta_2}$$

where $\theta_4 = -\frac{1}{\mu+\lambda}$

By standard forms of Laplace Transforms, we have

$$i = i_0 + (1 - \frac{(1-g)\beta}{\mu} + Be^{-\theta_1 t} + Ce^{-\theta_2 t})\,\bar{m}$$

Finally, from the subsidiary equation of (2c), we have

$$\frac{-(\beta\delta - g\beta\delta + \beta z)m}{z[\beta(\mu+\lambda)z^2 + Tz + \mu\delta]} = \frac{A(z+\theta_1)(z+\theta_2) + B(z+\theta_2)z + C(z+\theta_1)z}{z(z+\theta_1)(z+\theta_2)} \cdot m$$

by partial fractions.

Compare coefficients of both sides of the above equation:

$$A + B + C = 0$$

$$A\theta_2 + A\theta_1 + B\theta_2 + C\theta_1 = \frac{-1}{\mu+\lambda}$$

$$A\theta_1\theta_2 = \frac{g\beta\delta - \beta\delta}{\beta(\mu+\lambda)}$$

Since $\theta_1\theta_2 = \frac{\mu\delta}{\beta(\mu+\lambda)}$ Thus $A = \frac{-(1-g)\beta}{\mu}$

Substitute and solve for B and C:

$$B = \frac{\frac{-(1-g)\beta}{\mu}\theta_2 - \theta_4}{\theta_1 - \theta_2}$$

$$C = \frac{\theta_4 - \left(\frac{-(1-g)\beta}{\mu}\right)\theta_1}{\theta_1 - \theta_2}$$

Again, by standard forms of Laplace Transforms, we have

$$R = R_o + \left(\frac{-(1-g)\beta}{\mu} + Be^{-\theta_1 t} + Ce^{-\theta_2 t}\right) \bar{m}$$